Position Rectification Control for Mecanum Wheeled Omni-directional Vehicles

Pakpoom Viboonchaicheep, Akira Shimada, *Member, IEEE*, and Yuhki Kosaka

***Abstract*— In recent years, an omni-directional vehicle system using Mecanum wheels has been developed and used in wheelchairs. It should be more widely utilized as a kind of robot for intelligent wheelchairs or vehicle robots for hazards by making the best use of its unique features. This paper presents a position rectification method during position and orientation control with multi-sampling periods. The control system is based on unique kinematics, which are under holonomic constraints. The idea for the proposed rectification method consisting of symptomatic rectification and preventive rectification has been borrowed from medical science.**

***Keywords*—Mecanum, omni-dimensional, vehicle, control**

I. INTRODUCTION

NOWADAYS, Mecanum wheels, characterized by a special configuration, are anticipated for use in vehicles in order to improve their omni-directional capabilities [1][2]. Some examples of vehicle usage using their unique features are intelligent wheelchairs or vehicle robots in some hazards. In order to put this into practice, the following technologies are needed:
① Motion control technology that includes omni-directional control, collision avoidance, and shock avoidance for accidents,
②Management technology that includes environment recognition, motion learning, and motion planning,
③ Intelligent human interface technology.

In order to achieve these targets, motion control technology must be established so that electromechanical systems with multi-sensors can detect physical states with their own sampling periods. This paper introduces motion control technology for the system in sequence:First, the kinematics equation and dynamics equations related to Mecanum-wheel Omni-directional Vehicles should be derived [4]-[7]. Next, state expressions should be derived based on those two formulas and a motion controller designed [8]-[12]. The omni-directional vehicle consists of 4 Mecanum wheels that are set up on every side in the same direction like normal carts. They have three degrees of freedom: "x", "y", and "θ_z". The first two variables indicate position data and the last one indicates the orientation along the z-axis in a perpendicular direction. In actuality, since the vehicle has four wheels, it has four degrees of freedom. Therefore the vehicle has a redundant degree of freedom. In addition, each wheel is driven by an AC servomotor with a rotary encoder to detect the revolution angles of the rotors.

When the vehicle moves, the wheels often slip under the floor's various conditions. During such slips, the real position and orientation deviate from the planned course. The vehicle has two kinds of sensors: rotary encoders and another absolute position sensor such as a vision sensor. There is an assumption that the vehicle controller can detect the angle from rotary encoders in short periods and the absolute position and orientation information in long periods.

The word "ubiquitous" has attracted a great deal of attention recently. Today, necessary information can be obtained via a computer network anywhere at any time. Even if a vision sensor is not used, circumstances allow acquisition of accurate position and orientation information. Therefore, this paper assumes that the vehicle controller is able to acquire information without restrictions on the kinds of sensors.

The proposed position rectification methods are **Symptomatic Rectification,** which is a direct method by which the robot controller rectifies the position and orientation data gradually after every period of position detection by a vision sensor, and **Preventive Rectification,** an indirect method that renews the control parameters in order to avoid slips.

The idea for these two methods comes from the medical treatment for asthma attacks that consists of symptomatic treatment and preventive treatment [3] as noted by the second author during the raisin of two children. An example of the former is inhaled medicines that are used first because they start working quickly. The latter is used to prevent asthma attacks from starting.

II. KINEMATICS FOR OMNI-DIRECTIONAL VEHICLES

A. Vehicle Kinematics

Figure 1 shows the shape of a Mecanum wheel that consists of 16 free rollers with a 45-degree slope. Figure 2 shows the disposition of the wheels and the frames Σ_0, Σ_{iW} $(i = 1,2,3,4)$, respectively. $V_{iW}\,(i = 1,2,3,4) \in R$ is the velocity vector corresponding to wheel revolutions, where $V_{iW} = R_w \times \omega_{iW}$, R_w is the radius of wheel, ω_{iW} is the revolution velocity of the wheel, and $V_{ir}\,(i = 1,2,3,4) \in R$ is the tangential velocity vector of the free roller touching the floor.

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Fig. 1. Mecanum Wheel Configuration

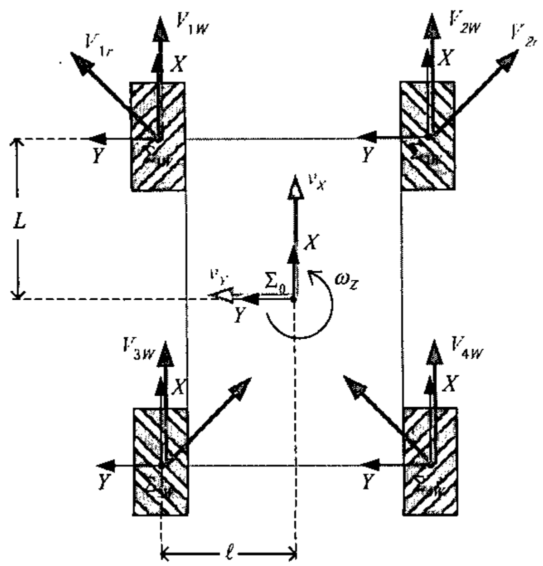

Fig.2. Disposition of Mecanum Wheels and Frames

V_{iX} is derived from the wheel velocity V_{iW} and $V_{ir}/\cos 45^\circ$. Similarly, V_{iY} is $V_{ir}/\sin 45^\circ$ from Fig. 2.

$$V_{1X} = V_{1W} + \frac{V_{1r}}{\sqrt{2}} \quad , \quad V_{1Y} = \frac{V_{1r}}{\sqrt{2}} \tag{1}$$

$$V_{2X} = V_{2W} + \frac{V_{2r}}{\sqrt{2}} \quad , \quad V_{2Y} = -\frac{V_{2r}}{\sqrt{2}} \tag{2}$$

$$V_{3X} = V_{3W} + \frac{V_{3r}}{\sqrt{2}} \quad , \quad V_{3Y} = -\frac{V_{3r}}{\sqrt{2}} \tag{3}$$

$$V_{4X} = V_{4W} + \frac{V_{4r}}{\sqrt{2}} \quad , \quad V_{4Y} = \frac{V_{4r}}{\sqrt{2}} \tag{4}$$

$$V_{1X} = v_X - \ell \cdot \omega_Z , \quad V_{1Y} = v_Y + L \cdot \omega_Z \tag{5}$$

$$V_{2X} = v_X + \ell \cdot \omega_Z , \quad V_{2Y} = v_Y + L \cdot \omega_Z \tag{6}$$

$$V_{3X} = v_X - \ell \cdot \omega_Z , \quad V_{3Y} = v_Y - L \cdot \omega_Z \tag{7}$$

$$V_{4X} = v_X + \ell \cdot \omega_Z , \quad V_{4Y} = v_Y - L \cdot \omega_Z \tag{8}$$

Meanwhile, v_X, v_Y, and $\omega_Z \in R$ represent the x and y elements of the velocity and angular velocity of the vehicle, respectively. In addition, V_{iX}, V_{iY} are expressed by using v_X, v_Y, and ω_Z above. By comparison with equations (1) – (4) and (5) – (8), the following equations are obtained:

$$V_{1W} = v_X - v_Y - (L + \ell)\omega_Z \tag{9}$$

$$V_{2W} = v_X + v_Y + (L + \ell)\omega_Z \tag{10}$$

$$V_{3W} = v_X + v_Y - (L + \ell)\omega_Z \tag{11}$$

$$V_{4W} = v_X - v_Y + (L + \ell)\omega_Z \tag{12}$$

Combining equations (9) – (12) into equation (13), which represents the inverse kinematics equation, yields:

$$V_W = J \cdot V_0 \tag{13}$$

where $V_O = [v_x, v_y, \omega_z]^T \in R^{3\times1}$ is the velocity vector in Cartesian coordinates; $V_w = [V_{1w}, V_{2w}, V_{3w}, V_{4w}]^T \in R^{4\times1}$ is the wheel velocity vector corresponding to the angular velocity.

$$J = \begin{bmatrix} 1 & -1 & -(L+\ell) \\ 1 & 1 & (L+\ell) \\ 1 & 1 & -(L+\ell) \\ 1 & -1 & (L+\ell) \end{bmatrix} \in R^{4\times3}$$

is a transformation matrix.

Oppositely, the vehicle velocity can be obtained from the wheel velocity using a pseudo inverse matrix as equation (14):

$$V_0 = J^+ \cdot V_w + (I - J^+ \cdot J)\varpi \tag{14}$$

where $J^+ = (J^T \cdot J)^{-1} J^T \in R^{3\times4}$. ϖ can be selected voluntarily. The radius of a wheel is R_w. Then, the forward kinematics equation is derived as follows at $\varpi = 0$.

$$\begin{bmatrix} v_X \\ v_Y \\ \omega_Z \end{bmatrix} = \frac{1}{4} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ -\frac{1}{(L+l)} & \frac{1}{(L+l)} & -\frac{1}{(L+l)} & \frac{1}{(L+l)} \end{bmatrix} \cdot \begin{bmatrix} R_w \cdot \dot{\theta}_1 \\ R_w \cdot \dot{\theta}_2 \\ R_w \cdot \dot{\theta}_3 \\ R_w \cdot \dot{\theta}_4 \end{bmatrix} \tag{15}$$

As a result, each element is given as follows:

$$v_X = \frac{R_w}{4}\left(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4\right) \tag{16}$$

$$v_Y = \frac{R_w}{4}\left(-\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 - \dot{\theta}_4\right) \tag{17}$$

$$\omega_Z = \frac{R_w}{4(L+l)}\left(-\dot{\theta}_1 + \dot{\theta}_2 - \dot{\theta}_3 + \dot{\theta}_4\right) \tag{18}$$

B. Vehicle Dynamics

The sum of the kinetic energy of the vehicle is given as equation (19), where "m" is the total mass of the vehicle." J_Z " is the vehicle moment of inertia around the Z axis. " J_W " is the wheel's moment of inertia around the center of revolution.

$$K = \frac{1}{2}\cdot m\cdot\left(V_X^2+V_Y^2\right)+\frac{1}{2}\cdot J_Z\cdot\omega_Z^2+\frac{1}{2}\cdot J_W\cdot\left(\dot\theta_1^2+\dot\theta_2^2+\dot\theta_3^2+\dot\theta_4^2\right) \tag{19}$$

Moreover, the loss energy is expressed in equation (20), where D_θ is the coefficient of the wheel's viscous friction.

$$D = \frac{1}{2}\cdot D_\theta\left(\dot\theta_1^2+\dot\theta_2^2+\dot\theta_3^2+\dot\theta_4^2\right) \tag{20}$$

Substituting equations (16) – (18) into equation (19) and utilizing Lagrangian equations yields equation (21).

$$\tau = M\ddot\theta + D_\theta\dot\theta \tag{21}$$

where $\theta := \begin{bmatrix}\theta_1 & \theta_2 & \theta_3 & \theta_4\end{bmatrix}^T \in R^4$,

$$\tau = \begin{bmatrix}\tau_1 & \tau_2 & \tau_3 & \tau_4\end{bmatrix}^T,$$

$$M=\begin{bmatrix} A+B+J_w & -B & B & A-B \\ -B & A+B+J_w & A-B & B \\ B & A-B & A+B+J_w & -B \\ A-B & B & -B & A+B+J_w \end{bmatrix} \in R^{4\times4}$$

$$A = \frac{m\cdot R^2}{8}, \qquad B = \frac{J_Z\cdot R^2}{16(L+l)^2}$$

III. Structure of the Velocity Control System for the Omni-directional Vehicle

A. Construction of Velocity Control

Based on equation (21), the state equation is yielded as:

$$\begin{bmatrix}\dot\theta \\ \ddot\theta\end{bmatrix} = \begin{bmatrix}0_{4\times4} & I_{4\times4} \\ 0_{4\times4} & -M^{-1}D_\theta\end{bmatrix}\begin{bmatrix}\theta \\ \dot\theta\end{bmatrix} + \begin{bmatrix}0_{4\times4} \\ M^{-1}\end{bmatrix}\tau \tag{22}$$

$$\dot\theta = \begin{bmatrix}0_{4\times4} & I_{4\times4}\end{bmatrix}\cdot\begin{bmatrix}\theta & \dot\theta\end{bmatrix}^T \tag{23}$$

In practice, the control algorithms are implemented in discrete form, so naturally the digital or sampled-data control law is used. As preliminary work, equations (22) and (23) are converted to equations (24) and (25) based on zero-order hold.

$$X[i+1] = A_d\cdot X[i] + B_d\cdot U[i] \tag{24}$$

$$Y[i] = C_d\cdot X[i] \tag{25}$$

where $X[i] = \left|\theta[i] \quad \dot\theta[i]\right|, U[i] = \tau[i], Y[i] = \dot\theta[i]$

The error between the reference velocity and real velocity is represented as $e[i] = Yr - Y[i]$; (26) is the integral of $e[i]$:

$$Z[i+1] = Z[i] + e[i] \tag{26}$$

Then, to design a digital servo system, the extended system is given by using equations (24) – (26).

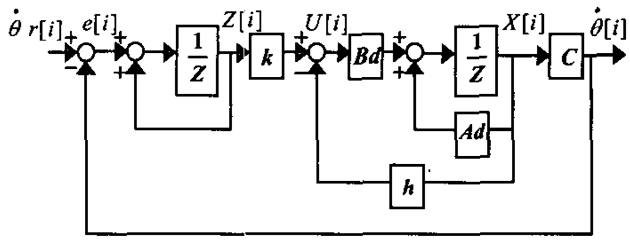

Fig.3. Mecanum wheel's velocity control system

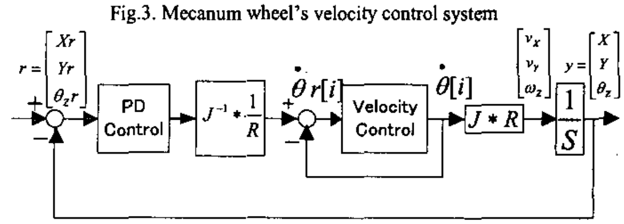

Fig.4. Structure of the Position Control System

$$\begin{bmatrix}X[i+1] \\ Z[i+1]\end{bmatrix} = \begin{bmatrix}A_d & 0_{8x4} \\ -C_d & I_{4x4}\end{bmatrix}\begin{bmatrix}X[i] \\ Z[i]\end{bmatrix} + \begin{bmatrix}B_d \\ 0_{4x4}\end{bmatrix}U[i] \tag{27}$$

The block diagram of the servo control system is shown in Fig. 3. The feedback gain vector *h* and integral gain *k* are given by the pole assignment method, LQ method, or the like.

$$U[i] = \begin{bmatrix}-h, & k\end{bmatrix}\cdot\begin{bmatrix}X[i] \\ Z[i]\end{bmatrix} \tag{28}$$

B. Construction of Position Control

As shown in Fig. 4, the position controller is designed to include the above velocity controller as an internal layer. The reason such a conventional structure is used rather than the latest controller structure is to carry out the position rectification by rending the motor velocity control portion independent via separation of the position control portion; the rectification will be discussed in the next section.

IV. Methods of Position Rectification

A. Symptomatic Position Rectification

It is assumed that data for the exact position and orientation is acquired from the vision sensor. Then, the error between the exact position data and the data from the rotary encoders can be rapidly calculated. Note that the sampling period for the vision sensor is longer than that for the encoder. Therefore, a short interval for the position rectification cannot be used.

To fix this problem, a new strategy is proposed where the controller gradually rectifies the position data by adding the position error to the position data detected from the rotary encoders through a low-pass filter in long sampling periods. The error data is acquired from the vision sensor via zero-order hold circuits [Fig. 5]. In order to readily evaluate the effect of the proposed method, results of a simulation are shown in Fig. 6

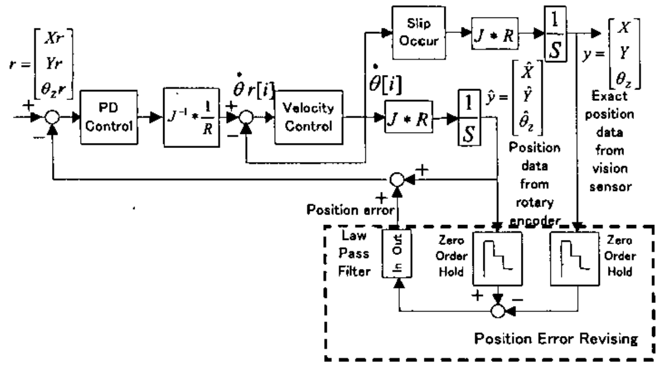

Fig.5. Control System with Symptomatic Rectification Block

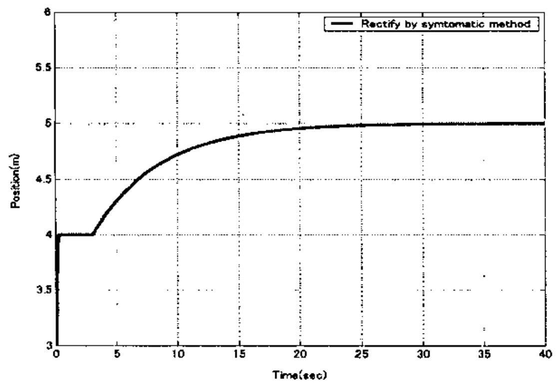

Fig.6. Step Response of the Proposed System with Symptomatic Rectification Block (Low-pass filter time of 4 seconds)

using a step input, where the vision sensor's sampling period is equal to 3 seconds. Notice that for the first 3 seconds, the position has not been rectified because of the lack of visual feedback. As can be noted, the effect of the gradual rectification is similar to an injection.

B. Preventive Position Rectification

The controller serves to estimate the real slip rate occurring on each wheel and adjust the control parameters so position error essentially does not occur. In Fig. 7, y is the real vehicle position signal from the vision sensor and $\hat{y}$ is the estimated vehicle position calculated from the rotary encoder; er is the position error. The result of the slip rate is given as $\beta = er/r$. In Fig. 7, there is a preventive rectification block, which includes the gain related to the slip rate. Receiving the information for the vehicle position error, the controller revises the control parameter, which is equal to $1-\beta$. Figure 8 shows the step response using both rectification methods. Note that the time decreases more than in Fig. 8. However, according to the simulation results, it has become clear that good time constant values of the low-pass filters depend heavily on the vision-sampling period.

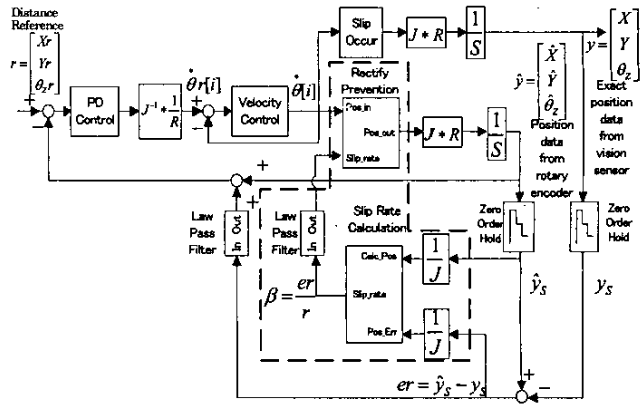

Fig.7. Control System with Both Rectification Blocks

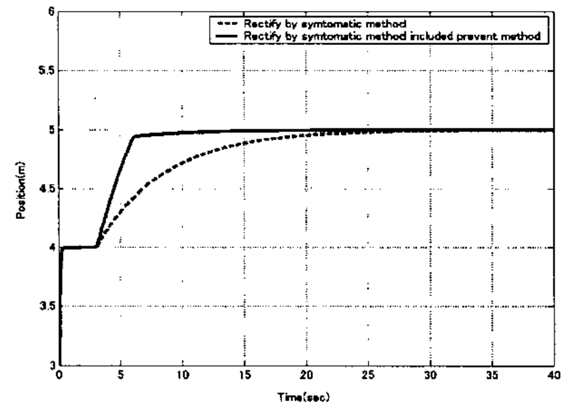

Fig.8. Step Response of the Proposed System with Both Prevention Blocks (Low-pass Filter time of 4 seconds)

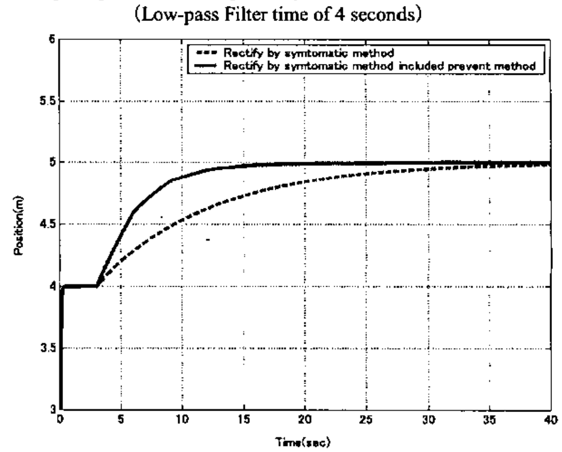

Fig.9. Step Response of the Proposed System with Both Prevention Blocks (Low-pass Filter time of 7 seconds)

C. Simulating by using Position Rectification Methods

This section simulates the vehicle motion according to the trajectory given as the benchmark in Fig.10. The vehicle should track smoothly. The reference trajectory given as $r(t) = [x_r(t) \quad y_r(t) \quad \theta_{zr}(t)]^T$ is described in global coordinates as the trajectory. Figure 10 shows the locus of vehicle motion as specified for it to reach its destination. The number in each figure of the vehicle means the order of the motion. From numbers 0 to5, the vehicle should change its orientation during motion. The orientation is equal to the tangential direction of the locus. However, on the way back to the start position, that is, from 5 to 9, the vehicle orientation should remain constant. Motion from 4 to 5 and from 9 to 10 consists only of rotations without transfer. Figure 11 shows the velocity specification related to numbers from 1 to 10. The upper portion describes tangential velocity and the lower portion indicates the angular velocity at the locus. Note that the trajectory means the locus with the constraint of the time. Figure 12 shows the reference position and orientation specification, which are equal to the integral of velocity specification. Figure 13 shows the simulation results for the Benchmark Reference with a Constant Slip Ratio. There is a slight positioning error during the motion.

Figure 14 shows an example of slip rates for wheels that are designed for evaluation of the proposed control system. In addition, Fig. 15 shows the simulation results for vehicle motion with the proposed rectification. The vehicle decreases the error gradually and smoothly although a slight error exists.

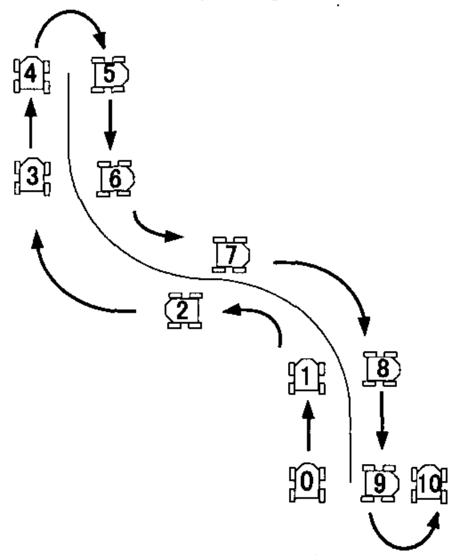

Fig.10. Locus Example as Benchmark

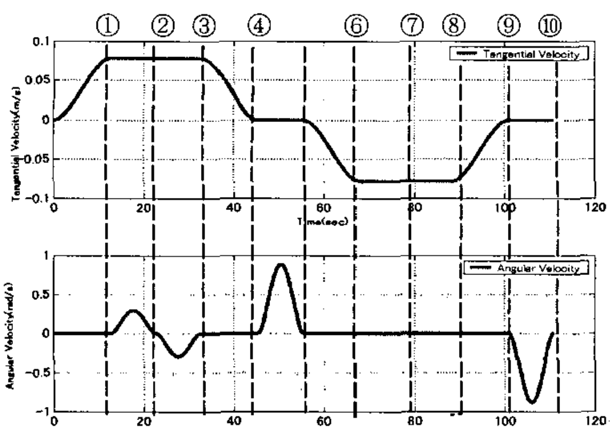

Fig.11. Tangential Velocity and Angular Velocity Specification

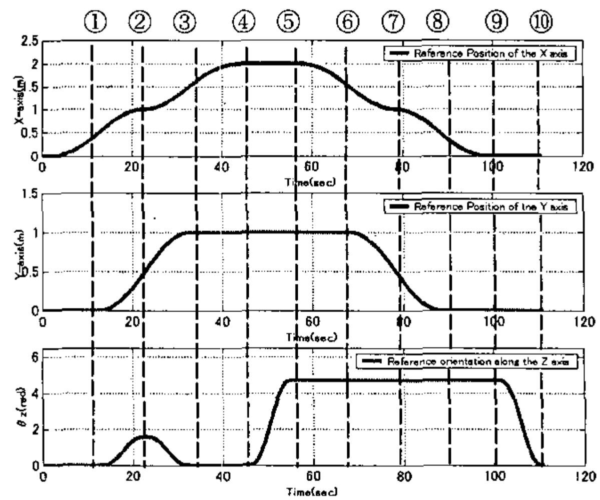

Fig.12. References for the Vehicle Control System

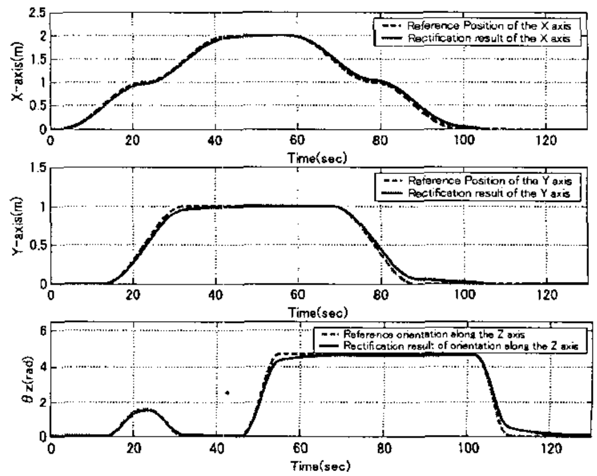

Fig.13. Simulation Results with a Constant Slip Ratio

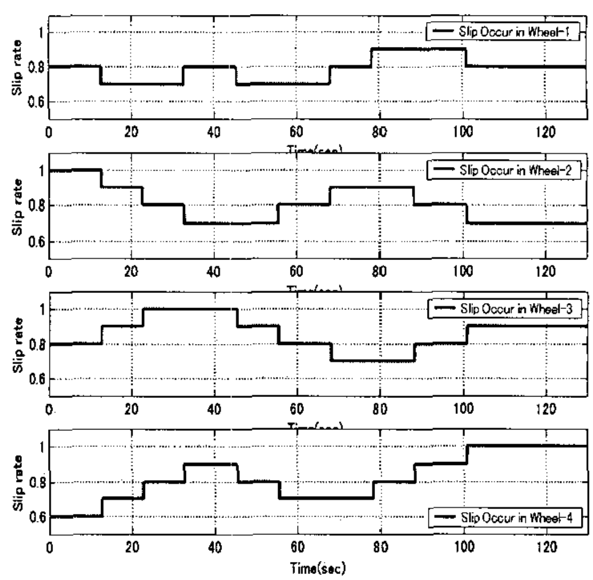

Fig.14. Example of Slip Rates during Motion

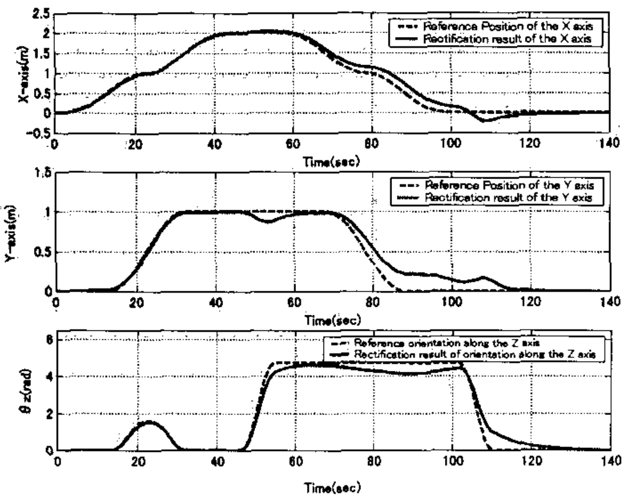

Fig.15. Simulation Results for Not-Constant Slip Rates

V. CONCLUSION AND FUTURE TOPICS

This paper has introduced a new method of position rectification for Mecanum-wheeled omni-directional vehicles. The method first includes the formulation of unique kinematics equation, dynamics equation, and state equation. Note that the vehicle functions to move anywhere in space with three degrees of freedom composing the horizontal plane, but it has four wheels, so the vehicle system has redundancy.

Next, a reference was proposed as a trajectory function in the same manner as a normal robot manipulator control system. Therefore, the control system needed its original forward and inverse kinematics calculation block.

Then, this paper introduced a method of handling a Mecanum-wheeled vehicle with uncertain slips occurring at the wheels.

The symptomatic rectification method, or the direct method, gradually and smoothly adjusts the error in position back to zero and was evaluated by a step response simulation.

The preventive rectification method, or the indirect method, disrupts the occurrence of the vehicle's positioning error and was also valuated. In addition, the final topic was evaluation of the total control system moving well along the designed reference as a benchmark.

In the future, this proposed method will be implemented in a real vehicle and evaluated in practice. Design of an optimal multi-sampling control system is considered to be important for an actual system.

Finally, the authors would like to thank Prof. Hisashi Takahashi, Polytechnic University, Japan, who lent us the mechanical base of the Mechanum wheelchair and who has encouraged us in these studies. Additionally, they would like to thank Dr. Takeshi Shinohara, administrator of the Shinohara Children's Clinic in Chiba Prefecture, Japan, who mentioned the current medical treatment for asthma attacks. His work was suggested the current methods proposed.

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